Table with the del operator in cylindrical, spherical and parabolic cylindrical coordinates

Operation	Cartesian coordinates (x,y,z)	Cylindrical coordinates (ρ,φ,z)	Spherical coordinates (r,θ,φ)	Parabolic cylindrical coordinates (σ,τ,z)
Definition of coordinates	$\rho = \sqrt{x^2+y^2}$ $\phi = \arctan(y/x)$ $z = z$	$x = \rho\cos\phi$ $y = \rho\sin\phi$ $z = z$	$x = r\sin\theta\cos\phi$ $y = r\sin\theta\sin\phi$ $z = r\cos\theta$	$x = \sigma\tau$ $y = \frac{1}{2}(\tau^2-\sigma^2)$ $z = z$
	$r = \sqrt{x^2+y^2+z^2}$ $\theta = \arccos(z/r)$ $\phi = \arctan(y/x)$	$r = \sqrt{\rho^2+z^2}$ $\theta = \arctan(\rho/z)$ $\phi = \phi$	$\rho = r\sin(\theta)$ $\phi = \phi$ $z = r\cos(\theta)$	$\rho\cos\phi = \sigma\tau$ $\rho\sin\phi = \frac{1}{2}(\tau^2-\sigma^2)$ $z = z$
Definition of unit vectors	$\hat{\boldsymbol\rho} = \frac{x}{\rho}\hat{\mathbf{x}} + \frac{y}{\rho}\hat{\mathbf{y}}$ $\hat{\boldsymbol\phi} = -\frac{y}{\rho}\hat{\mathbf{x}} + \frac{x}{\rho}\hat{\mathbf{y}}$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$\hat{\mathbf{x}} = \cos\phi\hat{\boldsymbol\rho} - \sin\phi\hat{\boldsymbol\phi}$ $\hat{\mathbf{y}} = \sin\phi\hat{\boldsymbol\rho} + \cos\phi\hat{\boldsymbol\phi}$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$\hat{\mathbf{x}} = \sin\theta\cos\phi\hat{\boldsymbol r} + \cos\theta\cos\phi\hat{\boldsymbol\theta} - \sin\phi\hat{\boldsymbol\phi}$ $\hat{\mathbf{y}} = \sin\theta\sin\phi\hat{\boldsymbol r} + \cos\theta\sin\phi\hat{\boldsymbol\theta} + \cos\phi\hat{\boldsymbol\phi}$ $\hat{\mathbf{z}} = \cos\theta\hat{\boldsymbol r} - \sin\theta\hat{\boldsymbol\theta}$	$\hat{\boldsymbol\sigma} = \frac{\tau}{\sqrt{\tau^2+\sigma^2}}\hat{\mathbf{x}} - \frac{\sigma}{\sqrt{\tau^2+\sigma^2}}\hat{\mathbf{y}}$ $\hat{\boldsymbol\tau} = \frac{\sigma}{\sqrt{\tau^2+\sigma^2}}\hat{\mathbf{x}} + \frac{\tau}{\sqrt{\tau^2+\sigma^2}}\hat{\mathbf{y}}$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$
	$\hat{\mathbf{r}} = \frac{x\hat{\mathbf{x}}+y\hat{\mathbf{y}}+z\hat{\mathbf{z}}}{r}$ $\hat{\boldsymbol\theta} = \frac{xz\hat{\mathbf{x}}+yz\hat{\mathbf{y}}-\rho^2\hat{\mathbf{z}}}{r\rho}$ $\hat{\boldsymbol\phi} = \frac{-y\hat{\mathbf{x}}+x\hat{\mathbf{y}}}{\rho}$	$\hat{\mathbf{r}} = \frac{\rho}{r}\hat{\boldsymbol\rho} + \frac{z}{r}\hat{\mathbf{z}}$ $\hat{\boldsymbol\theta} = \frac{z}{r}\hat{\boldsymbol\rho} - \frac{\rho}{r}\hat{\mathbf{z}}$ $\hat{\boldsymbol\phi} = \hat{\boldsymbol\phi}$	$\hat{\boldsymbol\rho} = \sin\theta\hat{\mathbf{r}} + \cos\theta\hat{\boldsymbol\theta}$ $\hat{\boldsymbol\phi} = \hat{\boldsymbol\phi}$ $\hat{\mathbf{z}} = \cos\theta\hat{\mathbf{r}} - \sin\theta\hat{\boldsymbol\theta}$	
A vector field $\mathbf{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_\rho\hat{\boldsymbol\rho} + A_\phi\hat{\boldsymbol\phi} + A_z\hat{\boldsymbol z}$	$A_r\hat{\boldsymbol r} + A_\theta\hat{\boldsymbol\theta} + A_\phi\hat{\boldsymbol\phi}$	$A_\sigma\hat{\boldsymbol\sigma} + A_\tau\hat{\boldsymbol\tau} + A_\phi\hat{\boldsymbol z}$
Gradient ∇f	$\frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$	$\frac{\partial f}{\partial \rho}\hat{\boldsymbol\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\boldsymbol\phi} + \frac{\partial f}{\partial z}\hat{\boldsymbol z}$	$\frac{\partial f}{\partial r}\hat{\boldsymbol r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol\phi}$	$\frac{1}{\sqrt{\sigma^2+\tau^2}}\frac{\partial f}{\partial \sigma}\hat{\boldsymbol\sigma} + \frac{1}{\sqrt{\sigma^2+\tau^2}}\frac{\partial f}{\partial \tau}\hat{\boldsymbol\tau} + \frac{\partial f}{\partial z}\hat{\boldsymbol z}$
Divergence $\nabla\cdot\mathbf{A}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$	$\frac{1}{\sigma^2+\tau^2}\frac{\partial A_\sigma}{\partial \sigma} + \frac{1}{\sigma^2+\tau^2}\frac{\partial A_\tau}{\partial \tau} + \frac{\partial A_z}{\partial z}$
Curl $\nabla\times\mathbf{A}$	$\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}}$ + $\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}}$ + $\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$	$\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\boldsymbol\rho}$ + $\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\boldsymbol\phi}$ + $\frac{1}{\rho}\left(\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right)\hat{\boldsymbol z}$	$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right)\hat{\boldsymbol r}$ + $\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi)\right)\hat{\boldsymbol\theta}$ + $\frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right)\hat{\boldsymbol\phi}$	$\left(\frac{1}{\sqrt{\sigma^2+\tau^2}}\frac{\partial A_z}{\partial \tau} - \frac{\partial A_\tau}{\partial z}\right)\hat{\boldsymbol\sigma}$ − $\left(\frac{1}{\sqrt{\sigma^2+\tau^2}}\frac{\partial A_z}{\partial \sigma} - \frac{\partial A_\sigma}{\partial z}\right)\hat{\boldsymbol\tau}$ + $\frac{1}{\sqrt{\sigma^2+\tau^2}}\left(\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right)\hat{\boldsymbol z}$
Laplace operator $\Delta f = \nabla^2 f$	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$	$\frac{1}{\sigma^2+\tau^2}\left(\frac{\partial^2 f}{\partial \sigma^2} + \frac{\partial^2 f}{\partial \tau^2}\right) + \frac{\partial^2 f}{\partial z^2}$
Vector Laplacian $\Delta\mathbf{A} = \nabla^2\mathbf{A}$	$\Delta A_x\hat{\mathbf{x}} + \Delta A_y\hat{\mathbf{y}} + \Delta A_z\hat{\mathbf{z}}$	$\left(\Delta A_\rho - \frac{A_\rho}{\rho^2} - \frac{2}{\rho^2}\frac{\partial A_\phi}{\partial \phi}\right)\hat{\boldsymbol\rho}$ + $\left(\Delta A_\phi - \frac{A_\phi}{\rho^2} + \frac{2}{\rho^2}\frac{\partial A_\rho}{\partial \phi}\right)\hat{\boldsymbol\phi}$ + $(\Delta A_z)\hat{\boldsymbol z}$	$\left(\Delta A_r - \frac{2A_r}{r^2} - \frac{2}{r^2\sin\theta}\frac{\partial(A_\theta\sin\theta)}{\partial\theta} - \frac{2}{r^2\sin\theta}\frac{\partial A_\phi}{\partial \phi}\right)\hat{\boldsymbol r}$ + $\left(\Delta A_\theta - \frac{A_\theta}{r^2\sin^2\theta} + \frac{2}{r^2}\frac{\partial A_r}{\partial \theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\phi}{\partial \phi}\right)\hat{\boldsymbol\theta}$ + $\left(\Delta A_\phi - \frac{A_\phi}{r^2\sin^2\theta} + \frac{2}{r^2\sin\theta}\frac{\partial A_r}{\partial \phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\theta}{\partial \phi}\right)\hat{\boldsymbol\phi}$	
Differential displacement	$d\mathbf{l} = dx\hat{\mathbf{x}} + dy\hat{\mathbf{y}} + dz\hat{\mathbf{z}}$	$d\mathbf{l} = d\rho\hat{\boldsymbol\rho} + \rho d\phi\hat{\boldsymbol\phi} + dz\hat{\boldsymbol z}$	$d\mathbf{l} = dr\hat{\mathbf{r}} + rd\theta\hat{\boldsymbol\theta} + r\sin\theta d\phi\hat{\boldsymbol\phi}$	$d\mathbf{l} = \sqrt{\sigma^2+\tau^2}d\sigma\hat{\boldsymbol\sigma} + \sqrt{\sigma^2+\tau^2}d\tau\hat{\boldsymbol\tau} + dz\hat{\boldsymbol z}$
Differential normal area	$d\mathbf{S} = dy\,dz\,\hat{\mathbf{x}}+$ $dx\,dz\,\hat{\mathbf{y}}+$ $dx\,dy\,\hat{\mathbf{z}}$	$d\mathbf{S} = \rho\,d\phi\,dz\,\hat{\boldsymbol\rho}+$ $d\rho\,dz\,\hat{\boldsymbol\phi}+$ $\rho\,d\rho d\phi\,\hat{\mathbf{z}}$	$d\mathbf{S} = r^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{r}}+$ $r\sin\theta\,dr\,d\phi\,\hat{\boldsymbol\theta}+$ $r\,dr\,d\theta\,\hat{\boldsymbol\phi}$	$d\mathbf{S} = \sqrt{\sigma^2+\tau^2}, d\tau\,dz\,\hat{\boldsymbol\sigma}+$ $\sqrt{\sigma^2+\tau^2}d\sigma\,dz\,\hat{\boldsymbol\tau}+$ $\sigma^2+\tau^2 d\sigma, d\tau\,\hat{\mathbf{z}}$
Differential volume	$dV = dx\,dy\,dz$	$dV = \rho\,d\rho\,d\phi\,dz$	$dV = r^2\sin\theta\,dr\,d\theta\,d\phi$	$dV = \left(\sigma^2+\tau^2\right)d\sigma d\tau dz,$

Non-trivial calculation rules:

1. $\operatorname{div}\ \operatorname{grad} f = \nabla \cdot (\nabla f) = \nabla^2 f = \Delta f$ (Laplacian)
2. $\operatorname{curl}\ \operatorname{grad} f = \nabla \times (\nabla f) = \mathbf{0}$
3. $\operatorname{div}\ \operatorname{curl} \mathbf{A} = \nabla \cdot (\nabla \times \mathbf{A}) = 0$
4. $\operatorname{curl}\ \operatorname{curl} \mathbf{A} = \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$ (using Lagrange's formula for the cross product)
5. $\Delta fg = f\Delta g + 2\nabla f \cdot \nabla g + g\Delta f$